GM TOOLBOX: QUICK START

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1. Purposes. The `gm_toolbox` contains Matlab functions I have been writing along the years. They are classified in 11 categories

- Bilinear forms
- Eigenvalues
- Interpolation
- Isotropic
- Linear systems
- Matrices
- Optimization
- Poly systems
- Precond
- Quadrature
- Utilities

They are briefly described below.

In total there are 1149 m-functions.

2. Bilinear forms. This folder contains functions to compute bounds or approximate values of bilinear forms $u^T f(A)v$ where A is a symmetric matrix. There are special functions for $f(A)$ equal to A^{-1}, $exp(A)$ and $\sqrt{A}$.

The methods are described in the book
G.H. Golub and G. Meurant, Matrices, moments and quadrature with applications, Princeton University Press, (2010).

3. Eigenvalues. This folder contains functions to compute eigenvalues of matrices. The subfolder Symmetric contains functions for using the Lanczos method, dqds and a divide and conquer algorithm for symmetric tridiagonal matrices

The subfolder Non symmetric contains functions to compute Ritz and harmonic Ritz values with Arnoldi, Lanczos and other methods as well as functions related to the localization of Ritz values in the complex plane.

4. Interpolation. The purpose of the functions in this folder is to compute "good" interpolation points in two dimensional domains, particularly the unit disk. See the paper
G. Meurant and A. Sommariva, On the computation of sets of points with low Lebesgue constant on the unit disk, Journal of Computational and Applied Mathematics, v 345 (2019), pp. 388-404.

It contains as well functions for computing the Thacher-Tukey rational interpolant in one dimension.

5. Isotropic. This folder contains functions to compute isotropic vectors, that is, vectors b satisfying $b^*Ab = 0$ for a given matrix A. See the paper
G. Meurant, The computation of isotropic vectors, Numerical Algorithms, v 60 n 2, (2012), pp. 193-204.

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6. Linear systems. This folder is the main part of the toolbox.

The subfolder Direct contains functions for computing Cholesky and LU factorizations of matrices. They are not as fast as Matlab built-in functions but they can be use for pedagogic purposes.

The subfolder Least squares contains functions for CGLS and LSQR.

The subfolder Non symmetric contains many functions for Krylov subspace iterative methods for solving nonsymmetric linear systems. There are many variants of GMRES with and without deflation. See the book
G. Meurant and J. Duintjer Tebbens, Krylov methods for nonsymmetric linear systems, Springer (2020).

The subfolder Symmetric contains many variants of the conjugate gradient method for solving symmetric positive definite linear systems. See the book
G. Meurant, The Lanczos and conjugate gradient algorithms, from theory to finite precision computations, SIAM (2006).

7. Matrices. This folder contains functions to generate some test matrices as well as mat-files to load matrices used in the examples.

8. Optimization. This folder contains functions for the Praxis optimization method of Richard Brent. This method does not need derivatives of the function to be optimized.

9. Poly systems. This folder contains functions for solving (small) polynomial systems of equations. It includes methods using Grobner bases as well as methods using moments.

10. Precond. This folder contains functions for generating preconditioners to be used by functions in the Linear systems folder. It includes approximate factorizations, approximate inverses and algebraic multigrid preconditioners with many variants.

11. Quadrature. This folder contains functions for computing quadrature rules and generating orthogonal polynomials. See the paper
G. Meurant and A. Sommariva, Fast variants of the Golub and Welsch algorithm for symmetric weight functions in Matlab, Numerical Algorithms, v 67 n 3 (2014), pp. 491-506.

12. Utilities. This folder contains many utility functions, including those used by other folders.